5/16/17 "Success is a journey, not a destination."-Ben Sweetland

HW: "Review Sheet Q4T2"
Test 2 on Thursday

AIM: How do we add probabilities?

Warm Up:

#1, 3, 4, 5 from June 2016 Regents Exam

1 When $b > 0$ and d is a positive integer, the expression $(3b)^{\frac{2}{d}}$ is equivalent to

$\frac{\text{Power}}{\text{Root}}$

(1) $\frac{1}{\left(\sqrt[d]{3b}\right)^2}$ (3) $\frac{1}{\sqrt{3b^d}}$

(2) $\left(\sqrt{3b}\right)^d$ (4) $\left(\sqrt[d]{3b}\right)^2$

3 Given i is the imaginary unit, $(2 - yi)^2$ in simplest form is

(1) $y^2 - 4yi + 4$ (3) $-y^2 + 4$

(2) $-y^2 - 4yi + 4$ (4) $y^2 + 4$

$(2-yi)(2-yi)$

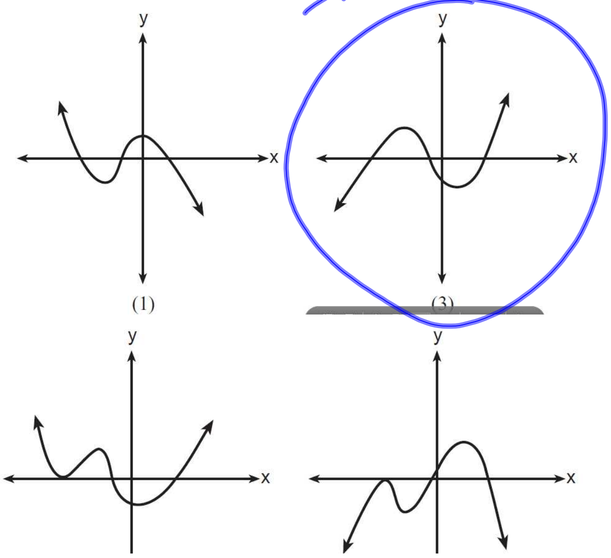
$4-2yi-2yi+y^2i^2$

$4-4yi-y^2$

$i^2 = -1$

4 Which graph has the following characteristics?

- three real zeros → hits x-axis 3 times
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$



5 The solution set for the equation $\sqrt{56 - x} = x$ is

(1) $\{-8,7\}$ (3) $\{7\}$

(2) $\{-7,8\}$ (4) $\{\ \}$

$(\sqrt{56-x})^2 = (x)^2$

$56 - x = x^2$

$-56 + x \quad +x - 56$

$0 = x^2 + x - 56$

$(x+8)(x-7)$

~~x=-8~~ $x=7$

Name: [redacted] Date: ____________

INTRODUCTION TO PROBABILITY
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Which of the following could *not* be the value of a probability? Explain your choice.

(1) 53% (3) $\frac{5}{4}$

(2) 0.78 (4) $\frac{3}{4}$

All probabilities must be numbers between 0 and 1. The number $\frac{5}{4} = 1.25$ and hence is not allowable.

(3)

2. If a month is picked at random, which of the following represent the probability its name will begin with the letter J?

(1) 0.08 (3) 0.12

(2) 0.25 (4) 0.33

$P(\text{beginning w/J}) = \frac{3}{12} = \frac{1}{4} = 0.25$

(2)

3. If a coin is tossed twice, which of the following gives the probability that it will land both times heads up or both times tails up?

(1) 0.75 (3) 0.25

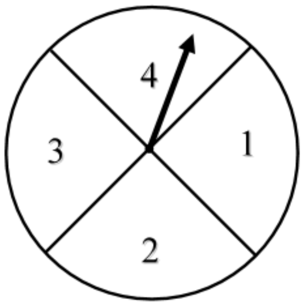
(2) 0.67 (4) 0.50

Sample Space:
$(H,H),(H,T),(T,H),(T,T)$

$P(\text{both heads or both tails}) = \frac{2}{4} = 0.50$

(4)

4. A spinner is now created with four equal sized sectors as shown. An experiment is run where the spinner is spun twice and the outcome is recorded each time.



(a) Create a sample space list of ordered pairs that represent the outcomes, such as $(4, 2)$, which represent spinning a 4 on the first spin and a 2 on the second spin.

Sample Space:
$(1,1),(1,2),(1,3),(1,4)$ $(2,1),(2,2),(2,3),(2,4)$
$(3,1),(3,2),(3,3),(3,4)$ $(4,1),(4,2),(4,3),(4,4)$

(b) Using your answer from (a), determine the probability of obtaining two numbers with a sum of 4.

Sum of 4: $(1,3),(3,1),(2,2)$

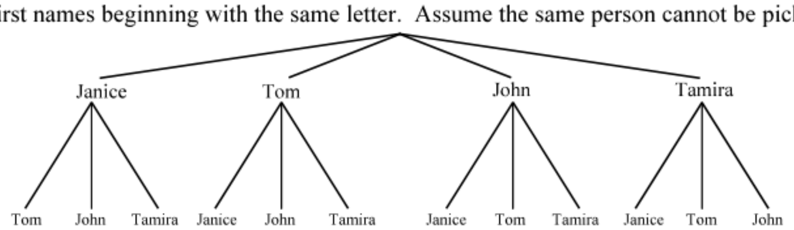
$P(\text{sum of 4}) = \frac{3}{16}$ **or 0.19 or 19%**

5. Samuel pulls two coins out of his pocket randomly without replacement. If his pocket contains one nickel, one dime, and one quarter, what is the probability that he pulled more than 20 cents out of his pocket? Justify your work by creating a tree diagram or a sample space.

(N, D)
$(N, Q)*$
(D, N)
$(D, Q)*$
$(Q, N)*$
$(Q, D)*$

$\boxed{P = \frac{4}{6} \text{ or } \frac{2}{3}}$

6. Janice, Tom, John, and Tamara are trying to decide on who will make dinner and who will wash the dishes afterwards. They randomly pull two names out of a hat to decide, where the first name drawn will make dinner and the second will do the dishes. Determine the probability that the two people pulled will have first names beginning with the same letter. Assume the same person cannot be picked for both.



$\boxed{P = \frac{4}{12} \text{ or } \frac{1}{3}}$

7. A blood collection agency tests 50 blood samples to see what type they are. Their results are shown in the table below.

Blood Type	Number of Samples
O	18
A	22
B	7
AB	3
Total	50

(a) If a blood sample is picked at random, what is the probability it will be type B?

$\boxed{P(\text{blood type B}) = \frac{7}{50} \text{ or } 0.14 \text{ or } 14\%}$

(b) If a blood sample is picked at random, what is the probability it will not be type O?

$\boxed{P(\text{not type O}) = \frac{22+7+3}{50} = \frac{32}{50} \text{ or } 0.64 \text{ or } 64\%}$

(c) Are the two probabilities you calculated in (a) and (b) **theoretical** or **empirical**? Explain your choice.

These are both empirical probabilities because they came from taking data. The actual probabilities people have these blood types could be quite different from these.

SETS AND PROBABILITY
COMMON CORE ALGEBRA II HOMEWORK

APPLICATION

1. Consider the experiment of picking one of the 12 months at random.

(a) Write down that sample space, S, for this experiment. What is the value of $n(S)$?

$$S=\begin{Bmatrix}\text{Jan, Feb, Mar, Apr, May, June}\\ \text{July, Aug, Sept, Oct, Nov, Dec}\end{Bmatrix}$$
$n(S)=12$

(b) Let E be the event (set) of picking a month that begins with the letter J. Write out the elements of E.

$E=\{\text{Jan, June, July}\}$

(c) What is the probability of E, i.e. $P(E)$?

$$P(E)=\frac{n(E)}{n(S)}=\frac{3}{12}=\frac{1}{4}\text{ or }0.25$$

(d) What is the probability of picking a month that does *not* start with the letter J?

$$P(\text{not }E)=\frac{n(\text{not }E)}{n(S)}=\frac{9}{12}=\frac{3}{4}\text{ or }0.75$$

2. Consider the set, A, of all integers from 1 to 10 inclusive (that means the 1 and the 10 are included in this set). Give a set B that is a subset of A. State its definition and list its elements in roster form. Then give a set C that is the complement of B.

Set B's Definition: **All integers divisible by 3 (for example).** Set B: $B=\{3,6,9\}$

Set C: $C=\{1,2,4,5,7,8,10\}$

3. If A and B are complements, then which of the following is true about the probability of B based on the probability of A?

(1) $P(B)=P(A)+1$ (3) $P(B)=\dfrac{1}{P(A)}$

(2) $P(B)=1-P(A)$ (4) $P(B)=P(A)-1$

$$P(B)+P(A)=1$$
$$-P(A)=-P(A)$$
$$P(B)=1-P(A)$$

(2)

4. If a fair coin is flipped three times, the probability it will land heads up all three times is $\frac{1}{8}$. Which of the following is the probability that when a coin is flipped three times at least one tail will show up?

(1) $\frac{7}{8}$ (3) $\frac{3}{2}$

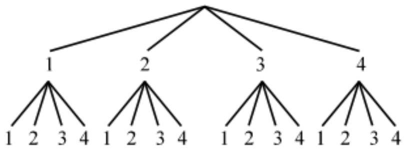
(2) $\frac{1}{8}$ (4) $\frac{1}{2}$

$$P(\text{not all heads})=1-\frac{1}{8}=\frac{7}{8}$$

(1)

5. A four-sided die, in the shape of a tetrahedron, is rolled twice and the number rolled is recorded each time.

(a) Draw a tree-diagram that shows the sample space, S, of this experiment. How many elements are in S?



16 total outcomes

(b) Let E be the event of rolling two numbers that have an odd product. List all of the elements of E as ordered pairs.

$$E = \{(1,1),(1,3),(3,1),(3,3)\}$$

(c) What is the probability that the two rolled numbers have a product that is odd?

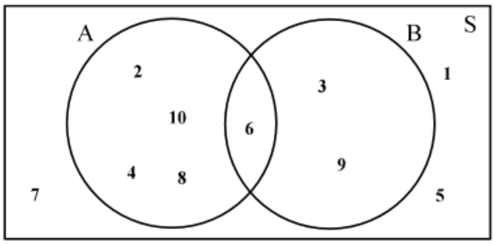
$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{16} = \frac{1}{4} \text{ or } 0.25$$

(d) What is the probability that the two rolled numbers have a product that is even?

$$P(\text{Even}) = 1 - P(\text{odd}) = 1 - \frac{1}{4} = \frac{3}{4} \text{ or } 0.75$$

REASONING

6. Consider the set of all integers from 1 to 10, i.e. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, to be our sample space, S. Let A be the set of all integers in S that are even and let B be the set of all integers in S that are multiples of 3. Fill in the circles of the Venn diagram with elements from S. If an element lies in both sets, place it in the overlapping region.



7. Find in the following:

$n(A) = \boxed{5}$ $\qquad$ $n(B) = \boxed{3}$

8. Why is the following equation *not* true? Explain.

$$n(S) = n(A) + n(B)$$

Two reasons that this equation fails to be true:

1. Not all the elements in *S* are in A or in B.

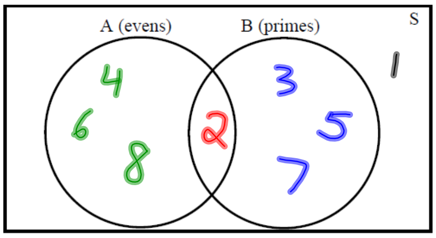
2. One element is in both A and in B.

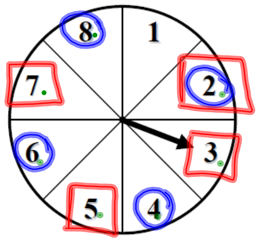
Fill in elements that are part of multiple sets first.

There are times that we want to determine the probability that either event A happened or event B happened. To do this, we need to be able to account for all of the outcomes that fall into either one of the two events. Let's see how this looks given a simple Venn diagram.

***Exercise* #1:** Consider the spinner shown below that has been divided into eight equally sized sectors of a circle. The spinner is spun once. In this experiment we will let A be the event of it landing on an even and B be the event of it landing on a prime number.

Fill in the Venn Diagram below with the actual numbers from the spinner.





When we have two (or more) sets, we can talk about their **union** and their **intersection**. Their technical definitions are given below.

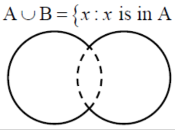
all overlap

THE UNION AND INTERSECTION OF TWO SETS

For two sets, A and B, their **union**, OR, and their **intersection**, AND, are given by:

(1) **Union**:

$A \text{ or } B = A \cup B = \{x : x \text{ is in A or } x \text{ is in B}\}$



(2) **Intersection:**

$A \text{ and } B = A \cap B = \{x : x \text{ is in A and } x \text{ is in B}\}$

***Exercise* #2:** From Exercise #1 write out the following two sets:

(a) A or B (The Union):

$A \cup B = \{2,3,4,5,6,7,8\}$

(b) A and B (The Intersection):

$A \cap B = \{2\}$

***Exercise* #3:** From Exercise #2, why is the equation $n(\text{A or B}) = n(\text{A}) + n(\text{B})$ generally *not* true? What would be the correct modification to make it true? Use the last example to help explain.

$n(A) = 4$

$n(B) = 4$

$n(A \text{ or } B) = 7$

$7 \neq 4+4$

Because "2" is counted twice

$$\boxed{n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)}$$

$7 = 4 + 4 - 1$ ✓

$\#\, A \text{ or } B = (\# \text{ in } A) + (\# \text{ in } B) - (\# \text{ in both})$

Two-way frequency charts give us a great example of how **events or sets can combine (union) and overlap (intersection).** Let's take a look at this and develop some ideas about probability along the way.

Exercise #4: A small high school surveyed 52 of its seniors about their plans after they graduate. They found the following data and wanted to analyze it based on gender. In this case, if we pick a student at random we can place them into one of four events:

M = Male F = Female

C = Going to College N = Not going to college

	Gender		
	Male	Female	Total
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

(a) Give the values for each of the following:

(i) $n(\text{M}) =$ 30 (ii) $n(\text{F}) =$ 22 (iii) $n(\text{C}) =$ 29 (iv) $n(\text{N}) =$ 23

(v) $n(\text{M and C}) =$ 16 (vi) $n(\text{F and C}) =$ 13 (vii) $n(\text{F or C}) =$ $22+29-13=\boxed{38}$

(b) What is the probability that a person picked at random would be a female who is going to college? Represent this using either a union or an intersection.

$\frac{\text{want}}{\text{total}} = \frac{13}{52} = \frac{1}{4} = .25$

25%

(c) What is the probability that a person picked at random would be a female or someone going to college? Represent this using either a union or an intersection.

$\frac{38}{52} = .73$

(d) Explain why $P(\text{F or C}) \neq P(\text{F}) + P(\text{C})$?

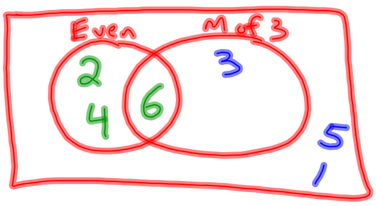
There are elements that are part of both sets.

(e) Fill in the general probability law based on (d):

$\boxed{P(\text{A or B}) = P(A) + P(B) - P(A \text{ and } B)}$

Sometimes we can avoid the probability law that we encounter in (e) by simply keeping careful track of what elements of the sample space are in both of our sets and making sure we don't count any element twice.

Exercise #5: A standard six-sided die is rolled once. Find the probability that the number rolled was either an even or a multiple of three. Represent this problem and the sets involved using a Venn diagram. Even though you don't need it, verify the **probability addition rule** from Exercise #4 (e).



$P(\text{Even or Mult of 3}) = \boxed{\frac{4}{6}} = \frac{2}{3}$

$P(\text{E or M of 3}) = P(E) + P(M) - P(\text{both})$

$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \boxed{\frac{4}{6}}$

There are some situations, though, where the **probability addition rule** is unavoidable.

Exercise #6: Insurance companies typically try to sell many different policies to the same customers. At one such company, 56% of all of the customers have car insurance policies, 48% have home insurance policies, and 18% have both. A customer is picked at random.

(a) Find the probability that she or he has at least one of the policies.

(Either/or)

$P(\text{C or H}) = P(C) + P(H) - P(\text{Both})$

$= .56 + .48 - .18$

$= .86$ $\boxed{86\%}$

(b) Find the probability that she or he has neither of the policies.

$P(\text{Neither}) = 1 - P(\text{Either})$

$= 1 - .86$

$= .14$

$\boxed{14\%}$

1. Given the two sets below, give the sets that represent their union and their intersection.

$$A = \{3, 5, 7, 9, 11, 13\} \qquad B = \{1, 5, 9, 13, 17\}$$

(a) Union: A or B =

(b) Intersection: A and B =

2. Using sets A and B from #1, verify the addition law for the union of two sets:

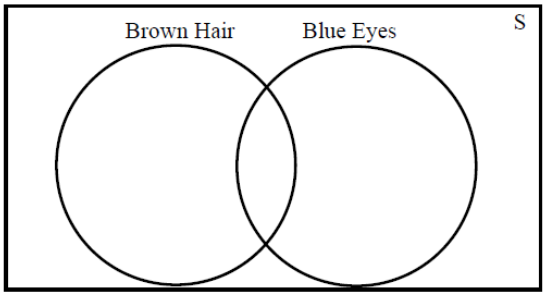
$$n(\text{A or B}) = n(\text{A}) + n(\text{B}) - n(\text{A and B})$$

3. Red Hook High School has 480 freshmen. Of those freshmen, 333 take Algebra, 306 take Biology, and 188 take both Algebra and Biology. Which of the following represents the number of freshmen who take at least one of these two classes?

(1) 639 (3) 451

(2) 384 (4) 425

4. Evie was doing a science fair project by surveying her biology class. She found that of the 30 students in the class, 15 had brown hair and 17 had blue eyes and 6 had neither brown hair nor blue eyes. Determine the number of students who had brown hair and blue eyes. Use the Venn Diagram below to help sort the students if needed.



5. A standard six-sided die is rolled and its outcome noted. Which of the following is the probability that the outcome was less than three or even?

(1) $\frac{2}{3}$ (3) $\frac{5}{6}$

(2) $\frac{1}{3}$ (4) $\frac{1}{6}$

______

6. Historically, a given day at the beginning of March in upstate New York has a 18% chance of snow and a 12% chance of rain. If there is a 4% chance it will rain and snow on a day, then which of the following represents the probability that a day in early March would have either rain or snow?

(1) 0.30 (3) 0.02

(2) 0.34 (4) 0.26

______

7. A survey was done of students in a high school to see if there was a connection between a student's hair color and her or his eye color. If a student is chosen at random, find the probability of each of the following events.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

(a) The student had black hair.

(b) The student had blue eyes.

(c) The student had brown eyes and black hair.

(d) The student had blue eyes or blond hair.

(e) The student had black hair or blue eyes.